Solutii clasa a VI-a

1)

$\frac{1\cdot y}{x\cdot y}+\frac{1\cdot x}{x\cdot y}=\frac{1}{p}$ => x$\cdot p=p(x+y)$ =>

(x-p)(y-p)=p^2

x$\neq y$ =>$\{x-p, y-p\}$={1, p^2} => {x,y} ={p+1, p^2+p} =>

=> x+y=p+1+p^2+p=p+1+p(p+1)=$(p+1)^2$=p.p

2)

a)

A=k^2; M={1,-1, k, -k}

1+ (-1) + k + (-k)=0

1·(-1)· k · (-k)= k^2

b)

A=p, p=nr. prim => M⊂D_p

D_p={1, -1, p, -p} nu exista M⊂D_p

c)

2016=$2^5\cdot3^2\cdot7$

M_1={-1, -2, -3, -7, 2, 3, 8}

(-1)+(-2)+(-3)+(-7)+2+3+8=0

(-1)·(-2)·(-3)·(-7)·2·3·8=2016.

M_2={-8, -6, 1, 6, 7}

(-8)+(-6)+1+6+7=0

3) <u>Lema</u> Daca n$\geq$ 4 atunci $3^n > 2^{n+1} + 22$

<u>Dem.</u> $2^{n+1}+22 < 2^{n+1}+32 \leq 2^{n+2}$, *oricare ar fi* $n \geq 4$

$2^{n+2}=2^{n-4}\cdot 2^6 < 3^{n-4}\cdot 3^4 = 3^n$ q.e.d.

$p_{2016}^{2016} < 2^{2017}+22$ =>(din lema) $p_{2016}=2$ =>

$p_1+p_2^2+...+p_{2015}^{2015}=2^{2016}+22$ =>(din lema) $p_{2015}=2$ => .... =>$p_4=2$=>

$p_1+p_2^2+p_3^3=2^4+22$ => $p_1+p_2^2+p_3^3=38$=>

I. $p_3=3$ => $p_1+p_2^2=11$ => $p_1=2$, $p_2=3$

$p_1=7$, $p_2=2$

II. $p_3=2$ =>$p_1+p_2^2=30$ => $p_1=5$, $p_2=5$ =>

S={(2,3,3,2,2,...,2); (7,2,3,2,2,...,2); (5,5,2,2,...,2)} .

4) Fie D$\in$ AB a.i. PC$\equiv$ PD (ca in figura)

=> A mijlocul lui [BD] => AM linie mijlocie in ΔBCD => $\widehat{DCQ} \equiv \widehat{ATQ}$=90°

($\measuredangle$ *corespondente*), unde AM $\cap$ $CQ=\{T\}$.

Ducem PN$\perp$ CD => [DN] $\equiv$ [NC], dar PN$\parallel$ CQ => PN linie mijlocie in ΔDCQ

=> P mijlocul lui DQ => DP$\equiv$ $PQ \equiv PC$

=> AQ + AP=PD=PC=AB-AP=AQ+BQ-AP => 2AP=BQ .