Solutii clasa a VIII-a

1)

$\sum \frac{a+b}{c^2} \geq \frac{9}{a+b+c} + \sum \frac{1}{c}$ <=> $\sum \frac{(a+b-c)(a+b+c)}{c^2} \geq 9$ <=> $\sum \frac{(a+b)^2-c^2}{c^2} \geq 9$ <=>
$\sum \frac{(a+b)^2}{c^2} \geq 12$

<=> $\sum(\frac{a^2}{c^2} + \frac{b^2}{c^2} + \frac{2ab}{c^2}) \geq 12$ <=> $\sum(\frac{a^2}{c^2} + \frac{c^2}{a^2}) + 2\sum \frac{ab}{c^2} \geq 12$.

Dar $\sum(\frac{a^2}{c^2} + \frac{c^2}{a^2}) \geq 3 \cdot 2$ iar $\sum \frac{ab}{c^2} \geq 3\sqrt[3]{\frac{a^2 \cdot b^2 \cdot c^2}{a^2 \cdot b^2 \cdot c^2}} = 1$ => ......$\geq 3 \cdot 2 + 2 \cdot 3 = 12$.

$\frac{9}{a+b+c} + \sum \frac{1}{a} \geq 4\sum \frac{1}{a+b}$ <=> $9 + \sum \frac{a+b+c}{a} \geq 4\sum \frac{a+b+c}{a+b}$ <=> $12 + \sum \frac{b+c}{a} \geq 4\sum \frac{c}{a+b}$

<=> $\sum \frac{b+c}{a} \geq 4\sum \frac{c}{a+b}$

Dar $4\sum \frac{c}{a+b} = \sum \frac{4}{\frac{a}{c}+\frac{b}{c}} \overset{\text{Titu}}{\leq} \sum(\frac{1}{\frac{a}{c}} + \frac{1}{\frac{b}{c}}) = \sum(\frac{c}{a} + \frac{c}{b}) = \sum(\frac{c}{a} + \frac{b}{a}) = \sum \frac{b+c}{a.}$.

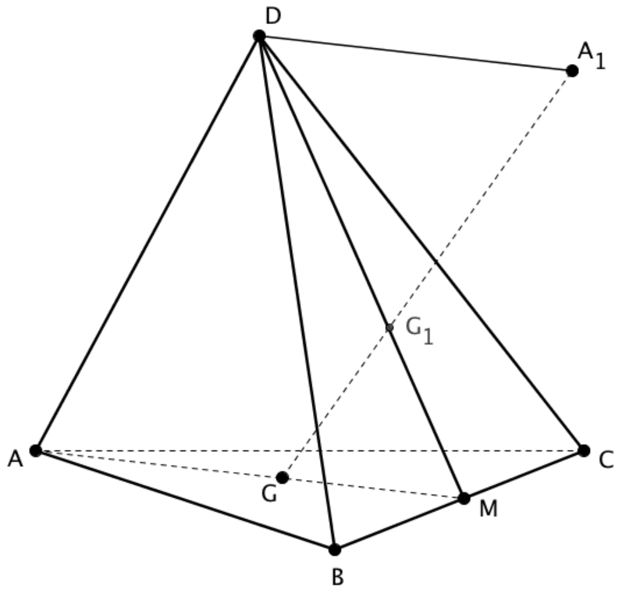
2) Pp. prin reducere la absurd afirmatia falsa => orice doi copii au un numar impar de prieteni. Fie P un copil, A multimea prietenilor sai, B complementara multimii A. Deoarece $|A|$ este par si numarul total de copii este 2016 =>$|B|$ este impar.

Fie Q un copil din B. Din definitia lui B => Q nu este prieten cu P.

Dar P si Q au un numar impar de prieteni comuni => Q are un numar impar de prieteni comuni cu P care se gasesc in A => Q are un numar impar de prieteni in B (deoarece are un numar par de prieteni).

Deoarece in B avem un numar impar de copii si fiecare copil are un numar impar de prieteni in B, daca facem suma numerelor de prieteni ai tuturor copiilor din B vom obtine un numar impar.

Dar aceasta suma reprezinta de doua ori numarul relatiilor de prietenie din B, adica un numar par => contradictie => pp. făcută este falsa => exista doi copii care au un numar par de prieteni comuni.



3) a) $DA_1 \parallel AG$, $AD \parallel GA_1$ => AGA_1D paralelogram => $DA_1=AG$

Analog $DB_1 \equiv BG$ deoarece $AG \parallel DA_1$, $BG \parallel DB_1$ => $\widehat{AGB} \equiv \widehat{A_1DB_1}$

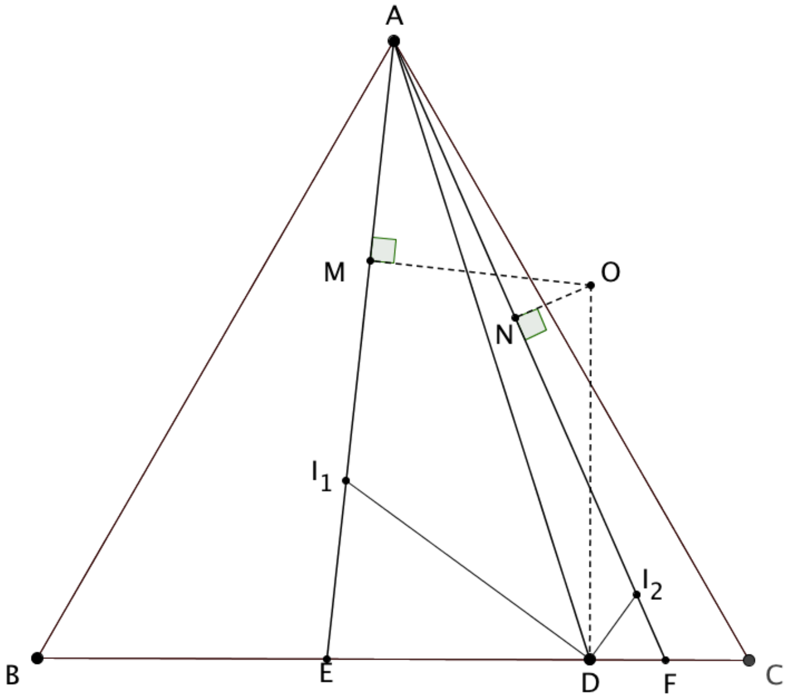
$DA_1 \equiv AG$, $BG \equiv DB_1$ => (L.U.L) $\Delta AGB \equiv \Delta A_1DB_1$ => $AB \equiv A_1B_1$.

Analog $AC \equiv A_1C_1$ si $BC \equiv B_1C_1$ => $\Delta ABC \equiv \Delta A_1B_1C_1$ si deoarece

$\Delta AGB \equiv \Delta A_1DB_1$

$\Delta AGC \equiv A_1DC_1$, $\Delta BGC \equiv B_1DC_1$ si G este centrul de greutate al $\Delta A_1B_1C_1$.

b) Deoarece $(ABC) \parallel (A_1B_1C_1)$ => cele doua tetraedre au inaltimile egale (=distante dintre plane) si deoarece $\Delta ABC \equiv \Delta A_1B_1C_1$ => $\frac{V_{\Delta ABC}}{V_{GA_1B_1C_1}} = 1$.



4) $\sphericalangle\ AI_2D = 90° + \frac{\hat{C}}{2} = 120°$ => AI_2DB inscriptibil.

Analog AI_1DC inscriptibil.

Aplicam lema lui Carnot este suficient sa demonstram ca

$AM^2+ED^2+FN^2=AN^2+DF^2+EM^2$. (unde $OM \perp AE$ si $ON \perp AF$) <=> $\frac{AI_1^2}{4} + ED^2 + (FI_2+\frac{AI_2}{2})^2 = \frac{AI_2^2}{4} + DF^2 + (EI_1^2 + \frac{AI_1}{2})^2$

<=> $ED^2+FI^2+FI_2 \cdot AI_2=DF^2+EI_1^2+EI_1 \cdot AI_1$

<=> $ED^2+FI_2 \cdot AF=DF^2+EI_1 \cdot AE$

<=> $ED^2+FD \cdot FB=DF^2+ED \cdot EC$

<=> $FD(FB-FD)=ED(EC-ED)$ <=> $FD \cdot BD=ED \cdot DC$

<=> $\frac{FD}{DC} = \frac{ED}{BD}$ <=> $\frac{FD}{FC} = \frac{ED}{EB}$ <=> $\frac{AD}{AC}=\frac{AD}{AB}$.