Barem orientativ clasa a VII-a

1) $n^3+3n^2-n-3=(n-1)(n+1)(n+3)$ (1)
n impar $\Rightarrow n-1,\ n+1,\ n+3$, 4 nr. pare consecutive
$\Rightarrow (n-1)(n+1)(n+3) \vdots 2^4$ (2p)
$$\left.\begin{aligned} n=M_6+1 \Rightarrow n-1=M_6 \\ n=M_6+3 \Rightarrow n+3=M_6 \\ n=M_6+5 \Rightarrow n+1=M_6 \end{aligned}\right\} \Rightarrow (n-1)(n+1)(n+3) \vdots 3 \text{ (3p)}$$
$(2^4,3)=1 \Rightarrow (n-1)(n+1)(n+3) \vdots 2^4\cdot 3$ (1p)

2) $a=\underbrace{111\ldots1}_{n}\underbrace{999\ldots9}_{k} \Rightarrow a+1=\underbrace{111\ldots1}_{n-1}\underbrace{200\ldots0}_{k}$
$\Rightarrow s(a)=n+9k;\ s(a+1)=n+1$
$aleg\ n=2012\,p-1 \Rightarrow s(a+1) \vdots 2012$
$(9,2012)=1 \Rightarrow (\exists)\ x,y\ \in \mathbb{Z}\ a.î.\ 9x+2012y=1$ (1p)
$\Rightarrow s(a)=n+9k=2012\,p-1+9k=2012p-9x-2012y+9k=$
$\Rightarrow 9(k-x)+2012(p-y)$
Iau $k=2012\,m+x, m\in\mathbb{N}$ a.î. $k>0$ și $p\in\mathbb{N}$ a.î. $p-y>0 \Rightarrow$ o infinitate de m și p

3) $\widehat{DQC}=\widehat{DPQ}+\widehat{DBC} \Leftrightarrow \widehat{DPQ}+\widehat{QDP}=\widehat{DPQ}+\widehat{DBC} \Leftrightarrow \widehat{QDP}\equiv\widehat{DBC}$
$\Leftrightarrow \Delta QDP \sim \Delta QBD \Leftrightarrow \frac{QD}{QP}=\frac{QB}{QD} \Leftrightarrow QD^2=QB\cdot QP$
$\Leftrightarrow QD^2=\frac{2}{3}BC\cdot\frac{BC}{3}=2DC^2$
$\Leftrightarrow OD^2=CQ^2+DC^2$
$\Leftrightarrow \hat{C}=90^0 \Leftrightarrow ABCD\ dreptunghi$

4) T. Menelaus în $\Delta ABP, M-I-D\ \Rightarrow \frac{MP}{MB}\cdot\frac{BD}{DA}\cdot\frac{AI}{IP}=1 \Rightarrow \frac{IP}{AI}\cdot\frac{MB}{MP}=\frac{BD}{DA}$.
Dar $\frac{BE}{EQ}=\frac{BC}{CQ}\ (T.bis.) \Rightarrow \frac{BE}{EQ}=\frac{BD}{DA} \Leftrightarrow \frac{IP}{AI}\cdot\frac{MB}{MP}=\frac{BC}{CQ} \Leftrightarrow \frac{IP}{AI}=\frac{2MP}{CQ}\ (BC=2MB) \Leftrightarrow \frac{IP}{AI}=\frac{2MP}{AC-AB}$
Dar $\frac{IP}{AI}=\frac{PC}{AC}=\frac{PB}{AB}=\frac{PC-PB}{AC-AB}=\frac{CM+PM-(BM-PM)}{AC-AB}=\frac{2MP}{AC-AB} \Rightarrow (x) \Rightarrow \frac{BD}{DA}=\frac{BE}{EQ}\xRightarrow{R.T.Thales} DE \parallel AC$