Soluții

Clasa a VII – a

1) $\underbrace{x^2-2xy+y^2}+\underbrace{y^2-2yz+z^2}=2\Rightarrow(x-y)^2+(y-2)^2=2$

$\Rightarrow x-y=\pm1 \qquad y-z=\pm1 \qquad \Rightarrow y=z\pm1$

$x=y\pm1=z\pm1\pm1\Rightarrow x+y+z=3z\pm1\pm1\pm1=2015$

$\Rightarrow 3z=2015\pm1\div1\pm1\in\{2012,\dots,2018\};3z\vdots3\Rightarrow$

$\Rightarrow 2015\pm1\pm1\pm1\vdots2\Rightarrow 3z=2016\Rightarrow z=672$

$\Rightarrow z=672, y=671; x=672$

2) *a) Dacă* $6-2\ productiv\ \Rightarrow 6(x_1+1)\dots(x_k+1)=(2x_1+1)\dots(2x_k+1)\Rightarrow par=impar\ (fals)$

b) $5=\frac{5}{3}\cdot\frac{5}{3}\cdot\frac{9}{5};\ 13=\frac{13}{7}\cdot\frac{17}{9}\cdot\frac{33}{17}\cdot\frac{21}{11}$

c) $2015=5\cdot13\cdot31$

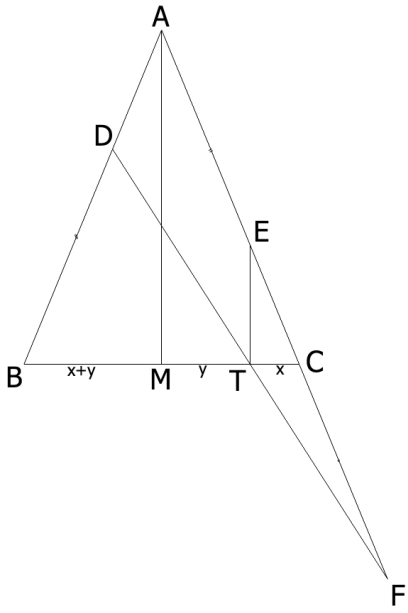
Rămâne să demonstrăm că 31 este q - productiv

$31=\frac{2\cdot30+1}{30+1}\cdot\frac{2^2\cdot30+1}{2\cdot30+1}\cdot\frac{2^3\cdot30+1}{2^2\cdot30+1}\cdot\frac{2^4\cdot30+1}{2^3\cdot30+1}\cdot\frac{2^5\cdot30+1}{2^4\cdot30+1}$

$\Leftrightarrow 31^2=32\cdot30+1\Leftrightarrow 31=(31+1)(31-1)+1\Leftrightarrow$

$31=31-1+1\Leftrightarrow 34^2=31^2$

3)



Ducem $AM\perp BC.$

Notăm $TC=x, MT=y\Rightarrow BM=x+y$

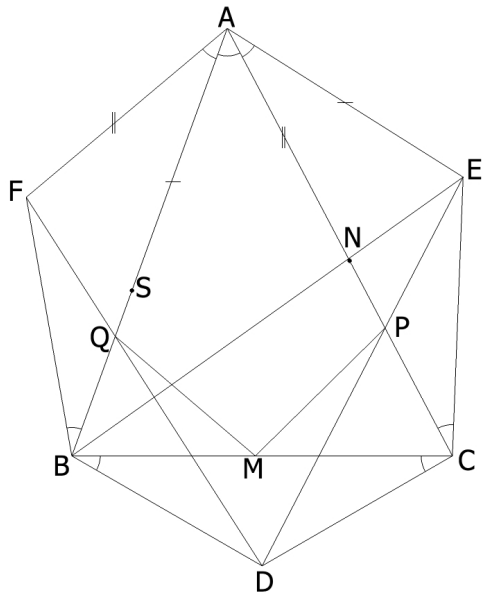
$\Delta ABC, D-T-F\ transversala$

$\overset{transversala}{\Longrightarrow}\frac{FC}{FA}\cdot\frac{AD}{DB}\cdot\frac{BT}{TC}=1\Rightarrow\frac{AF}{AD}=\frac{BT}{TC}$

$Dar\ AD=CE\Rightarrow\frac{AF}{CE}=\frac{TD}{TC}\Rightarrow\frac{AF-CE}{CE}=\frac{BT-TC}{TC}\Rightarrow\frac{2AE}{CE}=\frac{2y}{x}\Rightarrow$

$\frac{AE}{CE}=\frac{MT}{TC}\overset{RT}{\Longrightarrow}ET\parallel AM$ Dar $AM\perp BC\Rightarrow ET\perp BC$

4)



Fie $N \in (AC)$ *şi* $S \in (AB)$ *astfel* *încât* $[AN] \equiv [AF]$ *şi* $[AE] \equiv [AS]$

$\Delta\, AEC \,\mathbb{N}\, \Delta\, AFB (U.U) \Rightarrow \frac{AE}{AC} = \frac{AF}{AB} \Rightarrow$

$\frac{AE}{AN} = \frac{AC}{AB}$ $(AF \equiv AN)$; $\widehat{BAC} \equiv \widehat{NAE} \Rightarrow$

$\Rightarrow \Delta\, ABC \,\mathbb{N}\, \Delta\, ANE \Rightarrow \measuredangle\, ANE \equiv \measuredangle ABC \Rightarrow$

$m\,(\measuredangle\, ENC) = 180^0 - m\,(\measuredangle\, ABC) = m\,(\measuredangle BAC) + m\,(\measuredangle\, BCA) =$

$= m\,(\measuredangle BCA) + m\,(\measuredangle BCD) = m\,(\measuredangle ACD) \Rightarrow NE \parallel DC$ (1)

$$\left.\begin{array}{l}\Delta\, ANE \,\mathbb{N}\, \Delta\, ABC \Rightarrow \frac{AN}{NE} = \frac{AB}{BC} \\ \Delta\, AFB \,\mathbb{N}\, \Delta\, DCB \Rightarrow \frac{AF}{AB} = \frac{DC}{BC}\end{array}\right| \Rightarrow \frac{AN}{NE} = \frac{AF}{DC}$$

$Dar\ AN \equiv AF \Rightarrow NE \equiv DC$ (2)

Din 1 şi 2 $\Rightarrow CDNE\ paralelogram\ \Rightarrow P\ mijlocul\ lui\ (NC)$

$\Rightarrow PM\ l.m.\hat{\imath}n\ \Delta\, BEN \Rightarrow MP = \frac{BN}{2} = \frac{AN}{2}$

$analog\ MQ = \frac{AS}{2} \Rightarrow \frac{MP}{MQ} = \frac{AN}{AS} = \frac{AB}{AC}$